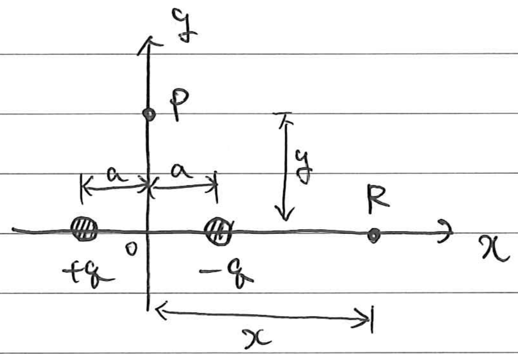
Lecture Note ⑧

e.x 1 The Electric Potential Due to Dipole



点Pの電気的Potential

$$V_P = k_e \sum_i \frac{q_i}{r_i} = k_e\left(\frac{q}{\sqrt{a^2+y^2}} + \frac{-q}{\sqrt{a^2+y^2}}\right) = 0$$

点Rの電気的Potential

$$V_R = k_e \sum_i \frac{q_i}{r_i} = k_e\left(\frac{-q}{x-a} + \frac{q}{x+a}\right) = -\frac{2k_e q a}{x^2-a^2}$$

x軸上の双極子から十分遠方のPotential

$$V_R = \lim_{x \gg a}\left(-\frac{2k_e q a}{x^2-a^2}\right) \approx -\frac{2k_e q a}{x^2}$$

a^2を無視する

負である。
-x方向の勾配

$$E_x = -\frac{dV}{dx} = -\frac{d}{dx}\left(-\frac{2k_e q a}{x^2}\right)$$

$$= 2k_e q a \frac{d}{dx}\left(\frac{1}{x^2}\right) = -\frac{4k_e q a}{x^3} \quad (x \gg a)$$

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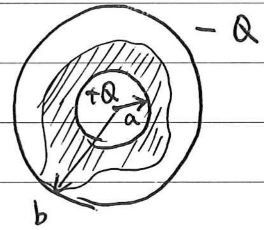
e.x 2. The Spherical Capacitor

半径 a, b の導体球殻がそれぞれ $-Q$, $+Q$に帯電している.



aとbの電位差 $V_b - V_a = -\int_a^b \mathbf{E} \cdot d\mathbf{s}$.

対称性より $\mathbf{E}$と$d\mathbf{s}$は平行かつ動径方向.

$$V_b - V_a = -\int_a^b E_r dr = -k_e Q \int_a^b \frac{dr}{r^2} = k_e Q \left[\frac{1}{r}\right]_a^b$$

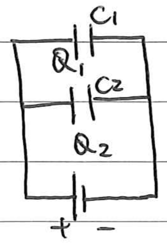
$$= k_e Q\left(\frac{1}{b} - \frac{1}{a}\right) = k_e Q \frac{a-b}{ab}$$

キャパシタンス $C = \frac{Q}{\Delta V} = \frac{Q}{V_b - V_a} = \frac{ab}{k_e(b-a)}$

※ここで $b \to \infty$とすると $C = \lim_{b\to\infty} \frac{ab}{k_e(b-a)} = \frac{ab}{k_e b} = \frac{a}{k_e} = 4\pi\varepsilon_0 a$.

孤立した導体球の電気容量

▶ コンデンサの並列接続

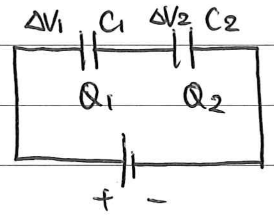


$$Q_{total} = Q_1 + Q_2$$

$$C_{eq} = C_1 + C_2$$

$\Leftarrow$ $Q = C\Delta V$

▶ コンデンサの直列接続



$$Q_1 = Q_2 = Q.$$

$$\Delta V_{total} = \Delta V_1 + \Delta V_2$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

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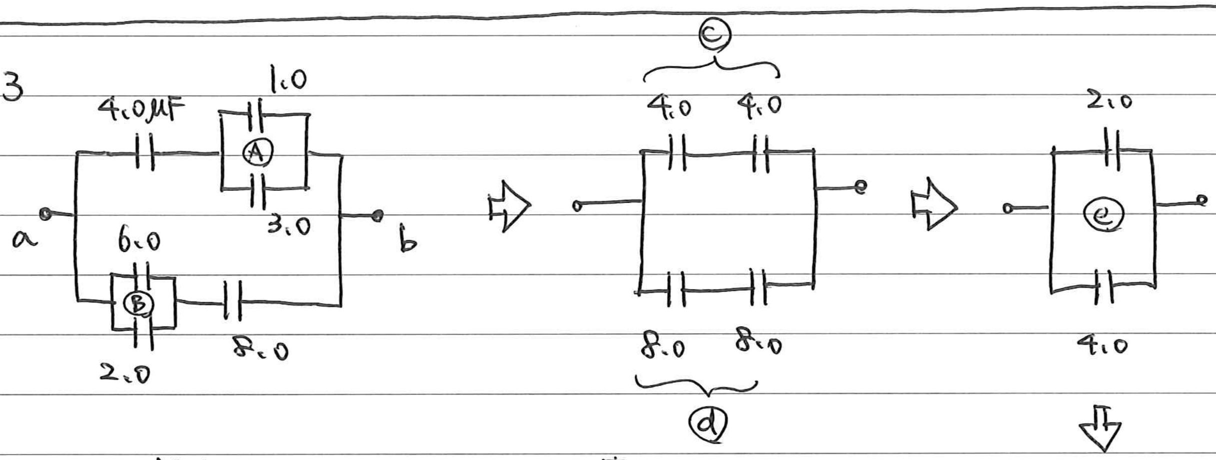
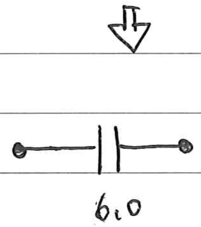
ex3





このように等価回路でキャパシタンスを見積る。

Ⓐ $C_{eq} = C_1 + C_2 = 4.0\,\mu F$

Ⓑ $C_{eq} = C_1 + C_2 = 8.0\,\mu F.$

ⓒ $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{4.0} + \frac{1}{4.0} = \frac{1}{2.0}$

ⓓ $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{8.0} + \frac{1}{8.0} = \frac{1}{4.0}$

ⓔ $C_{eq} = 2.0 + 4.0$

$= \boxed{6.0\,\mu F}$

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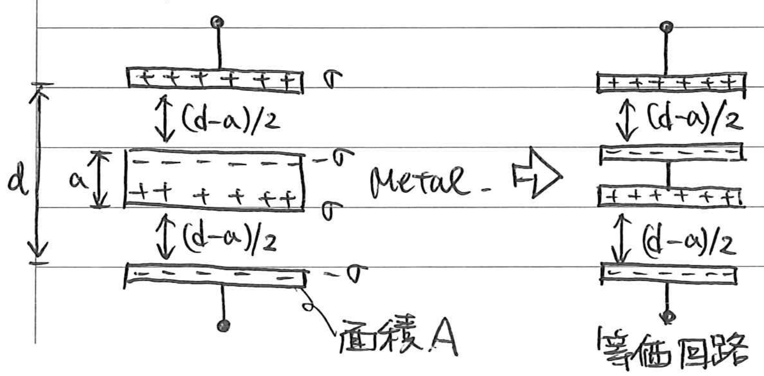
ex 4. Effect of Metallic Slab (真空中)



$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{\left[\frac{\varepsilon_0 A}{(d-a)/2}\right]} + \frac{1}{\left[\frac{\varepsilon_0 A}{(d-a)/2}\right]}$$

$$C = \frac{\varepsilon_0 A}{d-a}.$$